Pure Laziness: An Introduction to the Haskell Programming Language

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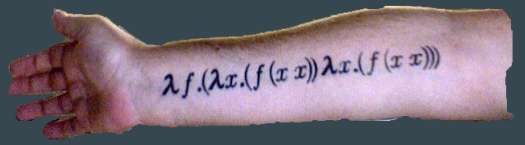
Richard's Route to Research



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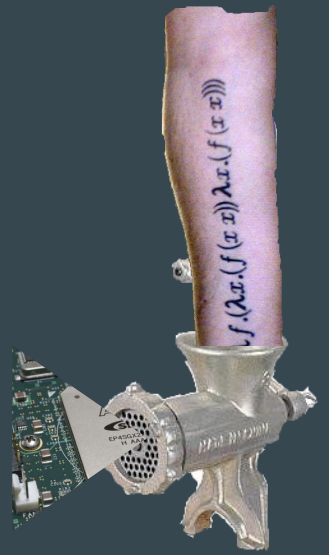

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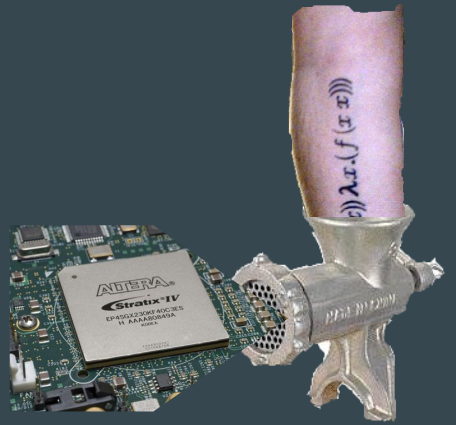


Richard's Route to Research

Bridging the abstraction gap

High-level

?

Programming Languages

Low-level

Python

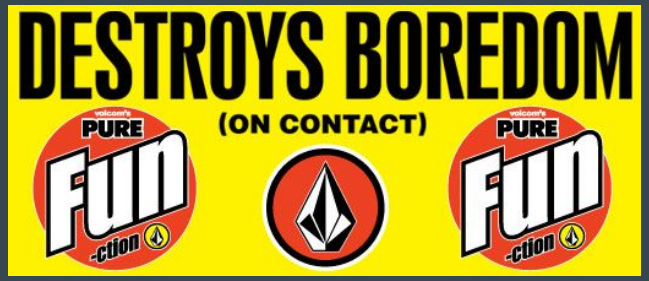
Java

C

Assembly

Haskell is Higher!

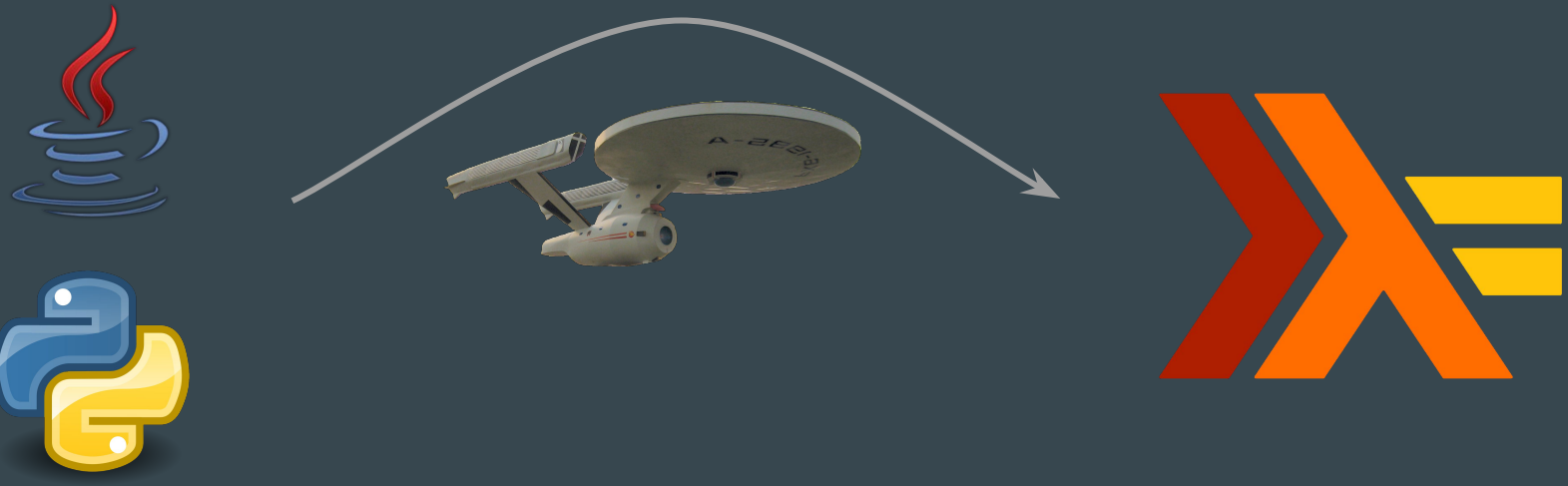
Pure functions



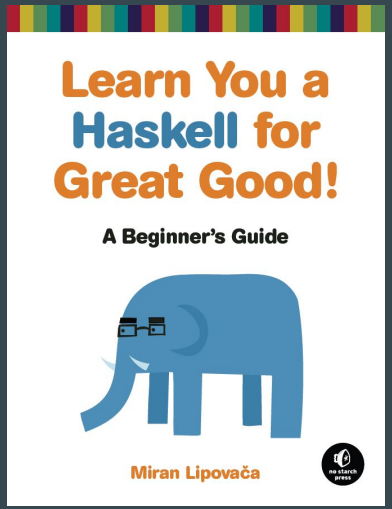
Powerful Type System

Lazy Evaluation

Exploring the Strange New World of Haskell



Exploring the Strange New World of Haskell



learnyouahaskell.com

Pure Functional Style

$f(x) = x^2 - 5$

$g(y) = f(4) + y$

```
f x = x^2 - 5

g y = f 4 + y
```

Pure Functional Style: Example

```
top3 doc = result
```

Input: A String representing a text document.

Output: A list of the 3 words that appear with highest frequency.

“In computer science functional programming is a programming paradigm a style of building the structure and elements of computer programs that treats computation as the evaluation of mathematical functions
and avoids changing state and mutable data It is a declarative programming paradigm which means programming is done with expressions”

http://www.onlamp.com/pub/a/onlamp/2007/07/12/introduction-to-haskell-pure-functions.html?page=

Pure Functional Style: Example

Input: A String representing a text document.

Output: A list of the 3 words that appear with highest frequency.

```
top3 doc = result
  where
    listOfWords = words doc
```

["In","computer","science","functional",
"programming","is","a","programming",
"paradigm","a","style","of","building","the",
"structure","and","elements","of",
"computer","programs","that","treats",
"computation","as","the","evaluation","of",
"mathematical","functions","and","avoids",
"changing","state","and","mutable","data",
"It","is","a","declarative","programming",
"paradigm","which","means",
"programming","is","done","with",
"expressions"]

http://www.onlamp.com/pub/a/onlamp/2007/07/12/introduction-to-haskell-pure-functions.html?page=

Pure Functional Style: Example

```
import Data.Char



top3 doc = result
  where
    listOfWords = words doc
    lowercase str = map toLower str
```

Input: A String representing a text document.

Output: A list of the 3 words that appear with highest frequency.

["In","computer","science","functional",
"programming","is","a","programming",
"paradigm","a","style","of","building","the",
"structure","and","elements","of",
"computer","programs","that","treats",
"computation","as","the","evaluation","of",
"mathematical","functions","and","avoids",
"changing","state","and","mutable","data",
"It","is","a","declarative","programming",
"paradigm","which","means",
"programming","is","done","with",
"expressions"]

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Pure Functional Style: Example

```
import Data.Char



top3 doc = result
  where
    listOfWords = words (lowercase doc)
    lowercase str = map toLower str
```

Input: A String representing a text document.

Output: A list of the 3 words that appear with highest frequency.

["in","computer","science","functional",
"programming","is","a","programming",
"paradigm","a","style","of","building","the",
"structure","and","elements","of",
"computer","programs","that","treats",
"computation","as","the","evaluation","of",
"mathematical","functions","and","avoids",
"changing","state","and","mutable","data",
"it","is","a","declarative","programming",
"paradigm","which","means",
"programming","is","done","with",
"expressions"]

http://www.onlamp.com/pub/a/onlamp/2007/07/12/introduction-to-haskell-pure-functions.html?page=

Pure Functional Style: Example

```
import Data.Char
import Data.List


top3 doc = result
  where
    listOfWords = words (lowercase doc)
    lowercase str = map toLower str
    wordGroups = sort listOfWords
```

Input: A String representing a text document.

Output: A list of the 3 words that appear with highest frequency.

["a","a","a","and","and","and","as","avoids","building","changing","computation","computer","computer","data","declarative","done","elements","evaluation","expressions","functional","functions","in","is","is","is","it","mathematical","means","mutable","of","of","of","paradigm","paradigm","programming","programming","programming","programming","programs","science","state","structure","style","that","the","the","treats","which","with"]

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Pure Functional Style: Example

```
import Data.Char
import Data.List


top3 doc = result
  where
    listOfWords = words (lowercase doc)
    lowercase str = map toLower str
    wordGroups = group (sort listOfWords)
```

Input: A String representing a text document.

Output: A list of the 3 words that appear with highest frequency.

```
[["a","a","a"],["and","and","and"],["as"],
["avoids"],["building"],["changing"],
["computation"],["computer","computer"],
["data"],["declarative"],["done"],
["elements"],["evaluation"],["expressions"],
["functional"],["functions"],["in"],
["is","is","is"],["it"],["mathematical"],
["means"],["mutable"],["of","of","of"],
["paradigm","paradigm"],
["programming","programming",
"programming","programming"],
["programs"],["science"],["state"],
["structure"],["style"],["that"],["the","the"],
["treats"],["which"],["with"]]
```

http://www.onlamp.com/pub/a/onlamp/2007/07/12/introduction-to-haskell-pure-functions.html?page=

Pure Functional Style: Example

```
import Data.Char
import Data.List
import Data.Ord

top3 doc = result
  where
    listOfWords = words (lowercase doc)
    lowercase str = map toLower str
    wordGroups = group (sort listOfWords)
    largestGroups = sortBy (comparing length) wordGroups
```

Input: A String representing a text document.

Output: A list of the 3 words that appear with highest frequency.

[["as"],["avoids"],["building"],["changing"],
["computation"],["data"],["declarative"],
["done"],["elements"],["evaluation"],
["expressions"],["functional"],["functions"],
["in"],["it"],["mathematical"],["means"],
["mutable"],["programs"],["science"],
["state"],["structure"],["style"],["that"],
["treats"],["which"],["with"],["computer",
"computer"],["paradigm","paradigm"],
["the","the"],["a","a","a"],["and","and",
"and"],["is","is","is"],["of","of","of"],
["programming","programming",
"programming","programming"]]

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Pure Functional Style: Example

```
import Data.Char
import Data.List
import Data.Ord

top3 doc = result
  where
    listOfWords = words (lowercase doc)
    lowercase str = map toLower str
    wordGroups = group (sort listOfWords)
    largestGroups = reverse (sortBy (comparing length) wordGroups)
```

Input: A String representing a text document.

Output: A list of the 3 words that appear with highest frequency.

[["programming","programming","programming","programming"],["of","of","of"],["is","is","is"],["and","and","and"],["a","a","a"],["the","the"],["paradigm","paradigm"],["computer","computer"],["with"],["which"],["treats"],["that"],["style"],["structure"],["state"],["science"],["programs"],["mutable"],["means"],["mathematical"],["it"],["in"],["functions"],["functional"],["expressions"],["evaluation"],["elements"],["done"],["declarative"],["data"],["computation"],["changing"],["building"],["avoids"],["as"]]

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Pure Functional Style: Example

```
import Data.Char
import Data.List
import Data.Ord

top3 doc = result
  where
    listOfWords = words (lowercase doc)
    lowercase str = map toLower str
    wordGroups = group (sort listOfWords)
    largestGroups = take 3 (reverse (sortBy (comparing length) wordGroups))
```

Input: A String representing a text document.

Output: A list of the 3 words that appear with highest frequency.

```
[["programming","programming",
"programming","programming"],["of","
of","of"],["is","is","is"]]
```

http://www.onlamp.com/pub/a/onlamp/2007/07/12/introduction-to-haskell-pure-functions.html?page=

Pure Functional Style: Example

```
import Data.Char
import Data.List
import Data.Ord

top3 doc = result
  where
    listOfWords = words (lowercase doc)
    lowercase str = map toLower str
    wordGroups = group (sort listOfWords)
    largestGroups = take 3 (reverse (sortBy (comparing length) wordGroups))
    result = map head largestGroups
```

Input: A String representing a text document.

Output: A list of the 3 words that appear with highest frequency.

["programming","of","is"]

http://www.onlamp.com/pub/a/onlamp/2007/07/12/introduction-to-haskell-pure-functions.html?page=

Powerful Type System: Explicit Types

```
listOfWords :: [String]
listOfWords = words (lowercase doc)
```

```
pythag :: Int -> Int -> Int -> Bool
pythag a b c = a^2 + b^2 == c^2
```

```
length :: [a] -> Int
length [] = 0
length (x:rest) = 1 + length rest
```

Powerful Type System: Define Your Own!

```
data Bool = True | False

data Shape = Square Float
           | Circle Float
           | Triangle Float Float Float

data Op = Add | Sub | Mul | Div

data Expr = Binop Expr Op Expr | Lit Int
```

Powerful Type System: Using your data types

```haskell
data Op = Add | Sub | Mul | Div
data Expr = Binop Expr Op Expr | Lit Int

eval :: Expr -> Int
eval (Lit x) = x
eval (Binop e1 Add e2) = eval e1 + eval e2
eval (Binop e1 Mul e2) = eval e1 * eval e2
eval (Binop e1 Sub e2) = eval e1 - eval e2
eval (Binop e1 Div e2) = eval e1 `div` eval e2
```

```haskell
eval (Lit 3)
=> 3
```

```haskell
eval (Binop (Lit 3) Add (Lit 4))
=> eval (Lit 3) + eval (Lit 4)
=> 3 + eval (Lit 4)
=> 3 + 4
=> 7
```

Executing Code

```
f :: Int -> Int -> Int
f x y = x + 1


f (27 + 2) (sum [1..10000000])
=> f 29 (sum [1..10000000])
=> ... computing sum ...
=> f 29 50000005000000
=> 29 + 1
=> 30
```

Laziness: I'll do it later...

```
f :: Int -> Int -> Int
f x y = x + 1


f (27 + 2) (sum [1..10000000])
=> (27 + 2) + 1
=> 29 + 1
=> 30
```

Laziness: To Infinity...

```
[1..10000000]
=> enumFromTo 1 10000000
=> 1 : enumFromTo (1 + 1) 10000000
=> 1 : 2 : enumFromTo (2 + 1) 10000000
=> … lots of calls …
=>  1 : 2 : … : 10000000 : []
```

```
enumFromTo :: Int -> Int -> [Int]
enumFromTo x y =
    if x > y
      then []
      else x : enumFromTo (x+1) y
```

```
[1..]
=> enumFrom 1
=> 1 : enumFrom (1 + 1)
=> 1 : 2 : enumFrom (2 + 1)
=> … lots of calls …
=>  1 : 2 : … : 10000000 : enumFrom (10000000 + 1)
=> … infinitely more calls …
```

```
enumFrom :: Int -> [Int]
enumFrom x = x : enumFrom (x+1)
```

Laziness: To Infinity...

```
take 2 [1..]
=> take 2 (enumFrom 1)
=> take 2 (1 : enumFrom 2)
=> 1 : take 1 (enumFrom 2)
=> 1 : take 1 (2 : enumFrom 3)
=> 1 : 2 : take 0 (enumFrom 3)
=> 1 : 2 : []
```

```
enumFrom :: Int -> [Int]
enumFrom x = x : enumFrom (x+1)

take :: Int -> [a] -> [a]
take 0 _  = []
take n [] = []
take n (x:xs) = x : take (n-1) xs
```

Laziness: ...and beyond!

```
zipWith :: (a -> b -> c) -> [a] -> [b] -> [c]
zipWith f [] _ = []
zipWith f _ [] = []
zipWith f (x:xs) (y:ys) = f x y : zipWith f xs ys
```

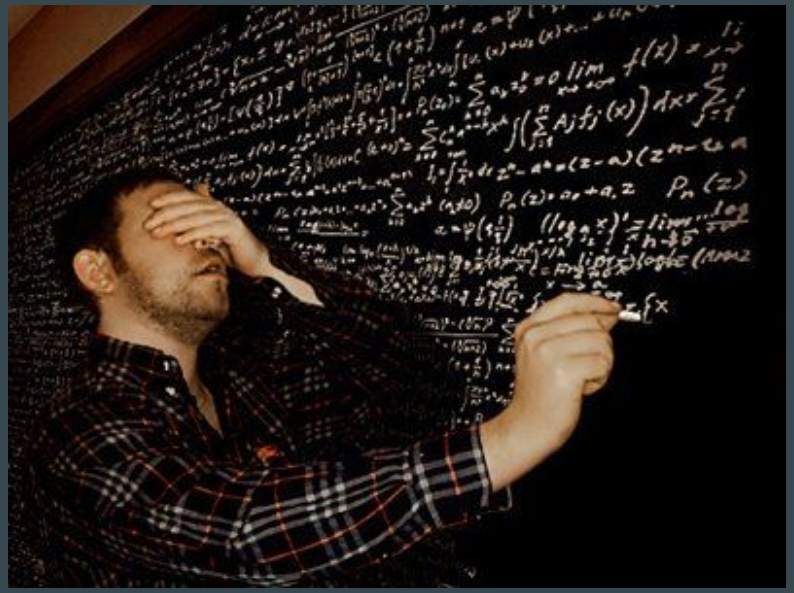
```
fibs = 1 : 1 : zipWith (+) fibs (tail fibs)
=> 1 : 1 : zipWith (+) (1:1:?) (1:?)
=> 1 : 1 : (1 + 1) : zipWith (+) (1:(1 + 1):?) ((1 + 1):?)
=> 1 : 1 : 2 : zipWith (+) (1:2:?) (2:?)
=> 1 : 1 : 2 : 3 : zipWith (+) (2:3:?) (3:?)
=> ...
```

```
fibs !! 0
=> 1
```

```
fibs !! 6
=> 13
```

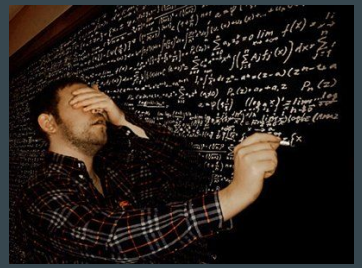
```
fibs !! 100
=> 573147844013817084101
```

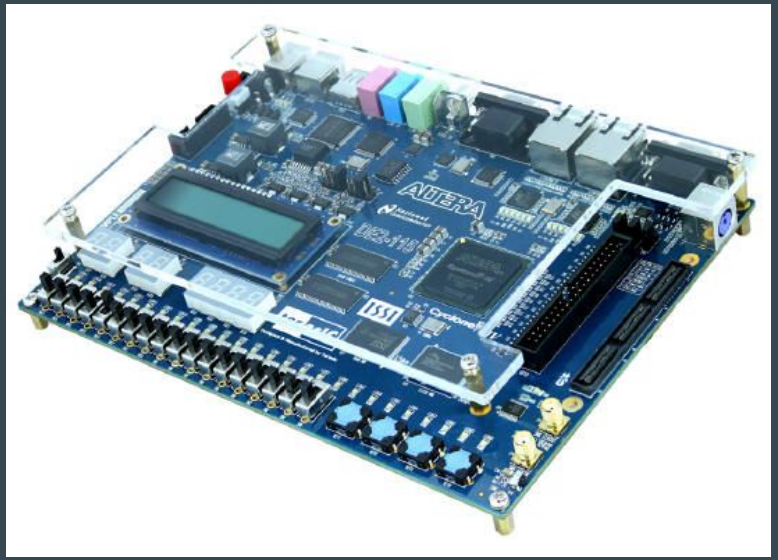
Proofs Are Trivial



$$
\begin{aligned}
&\texttt{unpack(pack(v))} \\
&= \texttt{unpack(pack(R p (R y z}^{k}\texttt{)}^{k}\texttt{))} \\
&= \texttt{unpack(P p (y (pack z)}^{k}\texttt{)}^{k}\texttt{)} \\
&= \texttt{R p (R y (unpack (pack z))}^{k}\texttt{)}^{k} \\
&= \texttt{R p (R y z}^{k}\texttt{)}^{k} \\
&= \texttt{v}
\end{aligned}
$$

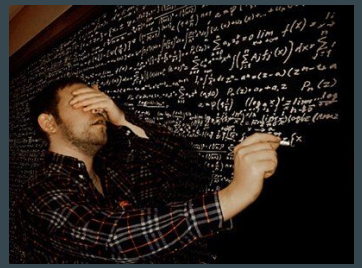
Embedding Languages

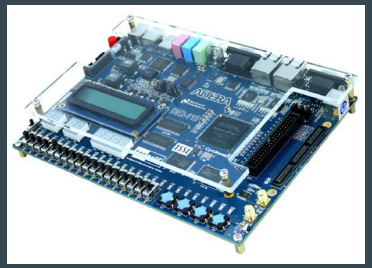


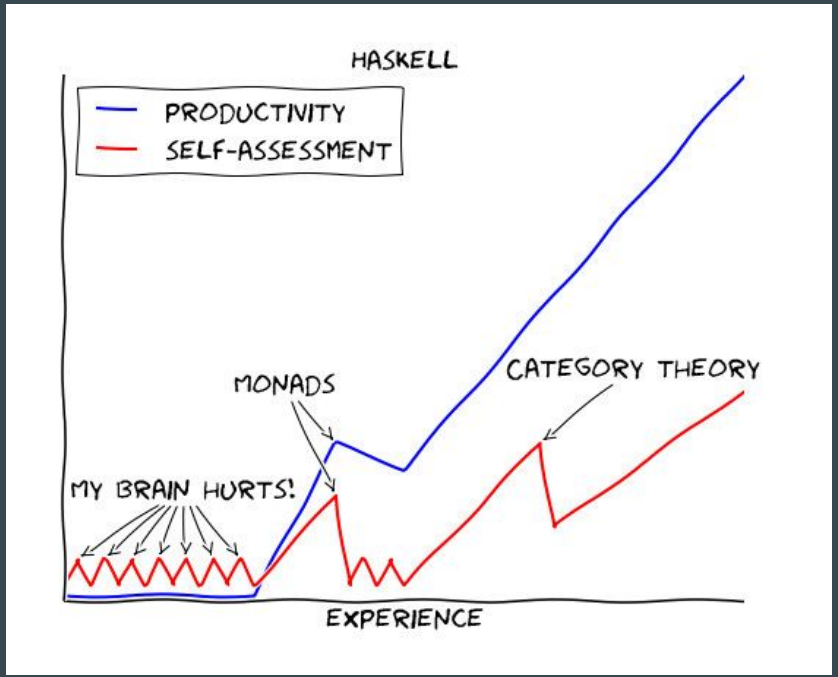



```
data Stream a = a :> (Stream a)

data MemOp addr word = MemRead addr
                     | MemWrite addr word


zipWith :: (a -> b -> c) ->
           Stream a -> Stream b -> Stream c
zipWith f (a :> as) (b :> bs) =
    f a b :> zipWith f as bs
```

Category Theory








```
class Applicative m => Monad m where

    (>>=)       :: forall a b. m a -> (a -> m b) -> m b

    (>>)        :: forall a b. m a -> m b -> m b
    m >> k = m >>= \_ -> k

    return      :: a -> m a
    return      = pure

    fail        :: String -> m a
    fail s      = error s
```

Thank you!